

Name: _____

Date: _____

Math 10F&PC Chapter 4 Roots and Powers**4.3 Mixed and Entire Radicals**

$$\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$$

entire
mixed

Expressing an **entire radical** as a **mixed radical** and vice versa is similar to the concept of writing equivalent fractions.

Just like $\frac{1}{2}$ is equivalent to $\frac{4}{8}$ the radical $\sqrt{8}$ is equivalent to $\sqrt{4 \cdot 2}$ equivalent to $2\sqrt{2}$.

And $\sqrt{25 \times 9}$ is the same as $\sqrt{25} \times \sqrt{9}$ because $5 \cdot 3 = 15$ $\sqrt{225} = 15$ $5 \times 3 = 15$ $\sqrt[3]{8} = \sqrt[3]{4 \cdot 2} = 2\sqrt[3]{2} \rightarrow \text{mixed}$

Entire Radical: a radical in the form $\sqrt[n]{x}$ Examples: $\sqrt{35}$, $\sqrt{68}$, $\sqrt[3]{47}$, $\sqrt[4]{39}$

Mixed Radical: a radical in the form $a\sqrt[n]{x}$ Examples: $3\sqrt{5}$, $-4\sqrt{21}$

The process for creating equivalent fractions is multiplying by a common number to both the top and bottom of the fraction. The process of writing equivalent radicals or writing an entire radical as a mixed radical, otherwise known as **simplifying** the radical involves the multiplication property of radicals:

$$\frac{3}{7} = \frac{3 \times 2}{7 \times 2} = \frac{6}{14} = \frac{6 \times 3}{14 \times 3} = \frac{18}{42} \dots$$

Multiplication Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b} \quad \text{Where } n \text{ is a natural number, and } a \text{ and } b \text{ are real numbers.}$$

$$\sqrt[3]{35} = \sqrt[3]{7 \cdot 5}$$

Part 1: Entire Radicals to Mixed Radicals

1. Find the **LARGEST** perfect square that is a factor of the radicand
2. Rewrite the radicand as a product of its largest square and another number.
3. Take the square root of the perfect square and write the answer as a mixed radical

Example 1:

a) $\sqrt{72} = \sqrt{36 \cdot 2} = 6\sqrt{2}$

b) $\sqrt{63} = \sqrt{9 \cdot 7} = 3\sqrt{7}$

c) $\sqrt[3]{108} = \sqrt[3]{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3} = \sqrt[3]{4 \cdot 27} = 3\sqrt[3]{4}$

d) $\sqrt[4]{128} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \sqrt[4]{16 \cdot 8} = 2\sqrt[4]{8}$

e) $\sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5} = 2 \cdot 2 \sqrt{5} = 4\sqrt{5}$

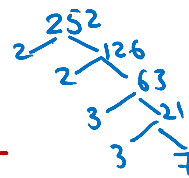
f) $\sqrt[4]{162} = \sqrt[4]{2 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = \sqrt[4]{81} = 3\sqrt[4]{2}$

Some number, such as 252, have more than one perfect square factor. The factors of 252 are: 1, 2, 3, 4, 6, 7, 9, 12, 14, 18, 21, 28, 36, 42, 63, 84, 126, 252. Since 4, 9, and 36 are perfect squares, we can simplify 252 in three ways.

$$\begin{aligned} \text{a) } \sqrt{252} &= \sqrt{4 \cdot 63} \\ &= 2\sqrt{63} \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{252} &= \sqrt{9 \cdot 28} \\ &= 3\sqrt{28} \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt{252} &= \sqrt{36 \cdot 7} \\ &= 6\sqrt{7} \\ &= \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 7} = \boxed{6\sqrt{7}} \end{aligned}$$



$6\sqrt{7}$ is in simplest form because the radical contains no perfect square factors other than 1.

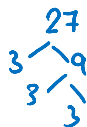
To write a radical of index n in simplest form, we write the radicand as a product of 2 factors, one of which is the greatest perfect n th power.

More mixed radicals that can be further reduced

$$\begin{aligned} \text{a) } 6\sqrt{27} &= 6\sqrt{3 \cdot 3 \cdot 3} \\ &= 6 \times 3\sqrt{3} = \boxed{18\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \text{b) } 4\sqrt{20} &= 4\sqrt{4 \cdot 5} \\ &= \boxed{8\sqrt{5}} \end{aligned}$$

$$\begin{aligned} \text{c) } 5\sqrt{80} &= 5 \times 4\sqrt{5} = \boxed{20\sqrt{5}} \end{aligned}$$



Part 2: Mixed Radicals to Entire Radicals

1. Square the number outside of the radical and write it under a radical sign.
2. Multiply the two numbers together and put the answer under a radical sign.

$$\begin{aligned} \text{a) } 3\sqrt{6} &= \sqrt{6 \cdot 3 \cdot 3} \\ &= \sqrt{54} \end{aligned}$$

$$\begin{aligned} \text{b) } 4\sqrt{5} &= \sqrt{5 \cdot 4 \cdot 4} \\ &= \sqrt{80} \end{aligned}$$

$$\begin{aligned} \text{b) } 6\sqrt{27} &= \sqrt{27 \cdot 6 \cdot 6} \\ &= \sqrt{972} \end{aligned}$$

$$\begin{aligned} \text{d) } 2^3\sqrt{2} &= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2} \\ &= \sqrt[3]{16} \end{aligned}$$

$$\begin{aligned} \text{e) } 5\sqrt{2} &= \sqrt{2 \cdot 5 \cdot 5} \\ &= \sqrt{50} \end{aligned}$$

$$\begin{aligned} \text{f) } 6^4\sqrt{5} &= \sqrt[4]{5 \cdot 6 \cdot 6 \cdot 6 \cdot 6} \\ &= \sqrt[4]{6480} \end{aligned}$$

Simplifying radicals. (Divide)

$$\begin{aligned} \text{a. } \frac{\sqrt{10}}{\sqrt{2}} &= \frac{\sqrt{2 \cdot 5}}{\sqrt{2}} \\ &= \frac{5\sqrt{2}}{\sqrt{2}} = \boxed{\sqrt{5}} \end{aligned}$$

$$\begin{aligned} \text{b. } \frac{\sqrt{6}}{\sqrt{2}} &= \frac{\sqrt{2 \cdot 3}}{\sqrt{2}} \\ &= \boxed{\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \text{c. } \frac{\sqrt{54}}{\sqrt{2}} &= \frac{\sqrt{2 \cdot 27}}{\sqrt{2}} \\ &= \sqrt{27} = \boxed{3\sqrt{3}} \end{aligned}$$

Assignment: p. 218 #4, 5, 9–12, 15–18, 20–22

Quiz next day